Physical resources to promote visualisation

Action research findings – Autumn 2011

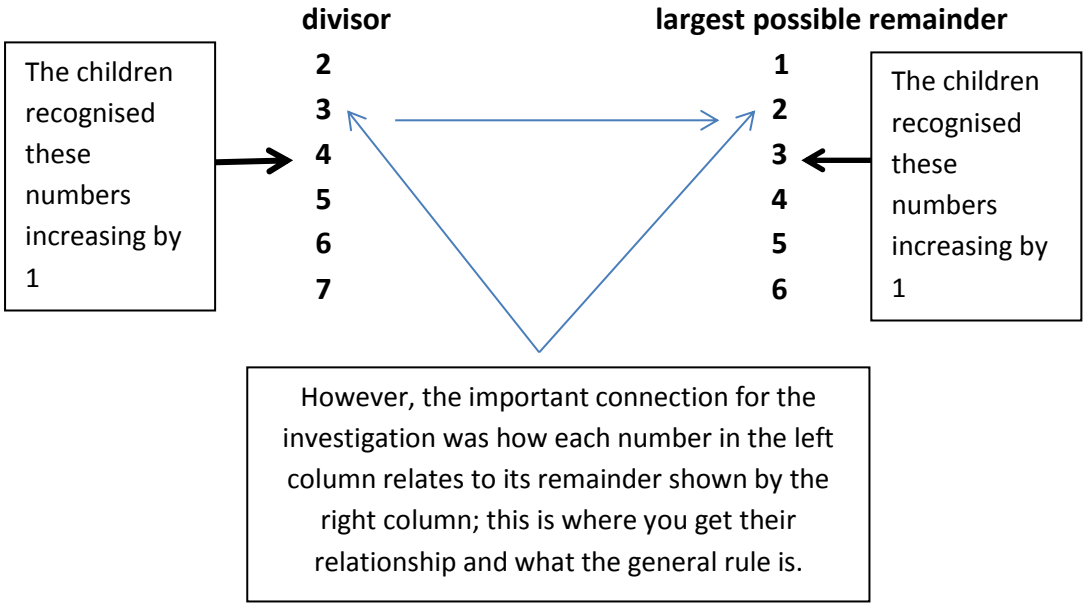
During the second half of the Autumn term, 2011, all seven schools and a maths consultant took part in action research sessions to answer the question: *'How can we use resources effectively to develop visualisation of number concepts in children?'*

Different groups of children, as well as a variety of number concepts, were looked at depending on what each school needed. Each of the days were hosted by a different school and representatives from two other schools visited each day this meant that each of the seven studies had three schools involved in it and that each school took part in three studies despite only having to host one of them.

General summary of findings:

- Resources need to be used by the children to develop their ability to visualise the concept therefore, if being used correctly they will wean themselves off the resources as their ability to visualise develops – the resources should not be used in a way that only helps them to get the answer
- They need to be using the equipment to investigate and 'discover' rather than just watching the teacher using the equipment/images
- Use of different and appropriate equipment needs thinking about i.e. use of hundred bead string to model taking a small number away from a larger number going over boundaries i.e. 72-8 (the alternate colours of the bead string help to picture the use of number bonds of ten and bonds of the number being subtracted) as well as bundles of items (i.e. straws in tens and ones or as units and tenths) where a bundle of ten needs to be 'turned' into ones; colours also need to be considered, i.e. whether they are all the same or a variety of colours
- Ensure that the resources are varied from the beginning so that limited models do not limit understanding and cause later confusions
- Children need to explore how things which are not exactly the same have similarities
- Use of different equipment and when each piece is appropriate needs thinking about i.e. use of hundred bead string to count on and back in tens from any number (the alternate colours of the bead string help to picture the use of number bonds of ten) and how this holds a different challenge to counting on and back in tens and ones using bundles or base ten blocks
- Persevere when using resources with children (when the novelty wears off they will use them appropriately)

- Don't take the resources away from the children, always have them available on the table and the children will choose to use them if needed (once they can visualise the concept they are working on they won't need the concrete representation of it)
- Teach more subtraction than addition and more division than multiplication etc.
- Use guided group work with children who are stuck to know the fine details of children's confusions and to address those confusions
- Children need to explore how things which are not exactly the same have similarities (not to focus on the differences) as this helps with recognising and explaining about general rules
- Encourage 'think, pair, share' so that children come to their own conclusion before they discuss approaches to hopefully lessen peer pressure that takes them 'down the wrong track'
- When asking the children to find patterns when investigating, ensure they focus on the pattern which is important i.e. when we did an investigation about finding the largest remainder for each divisor the children only focused on the pattern which was not relevant (see below)



When any concept is first introduced, consider how you can make it 'concrete'. Then the children should take part in activities that develop their ability to fully imagine the concept using the equipment, during the teaching input. The children continue to use the equipment to scaffold the follow-up independent activity. The equipment should continue to be available to be used by the children when they are working over subsequent lessons that build on extending and using the concept. To help with this, the resources, once introduced, are kept in a basket on the table to allow access during the lessons at any time. When the concept is revisited, possibly as O&M starters within other Units/lessons, the equipment is reintroduced and used by the children. When the concept of the objective is being used

through another area of mathematics then the resources are also reintroduced and available to be used by the children.

Concepts looked at during this study and how to teach them in a concrete form using resources:

Place Value and number order:

Children need to develop a 'feel' for the size of numbers and this cannot be achieved using numerals alone.

- Children's ability to 'subitise' needs work on to help with not counting from the beginning each time and also with recognising amounts
- Counting and adding on in tens and ones using towers of cubes and single cubes and also using a hundred bead-string as well as the Counting Cats (adapting the Counting Cats game to practise adding and subtracting 9 and 11)
- More use of resources with children to help them really understand how counting works and what it means (particularly with counting over boundaries and teens numbers including HTO as well as decimal/whole number boundaries, etc.)
- Building numbers using cubes, base 10, bead-string, straws in bundles of ten and single straws will help children compare their relative sizes and support with explaining
- A grid to place the base ten equipment on which models the principle of only up to 9 in each column can be used in conjunction with base 10 apparatus (the ITP, 'Beadstring 9' can also help to model this idea)
- Numbers should be taught and discussed in 4 separate ways: its name, how we write it, the physical representation of the amount, how to describe it i.e. forty-seven has 4 tens and 7 ones
- Activities where you are asking the children to write a number through listening to its description that is not necessarily in size order i.e. 'Write a number with 5 ones, 3 hundreds and 7 tenths in it', is a powerful way to promote a deeper understanding of how numbers are constructed (the children enjoy giving these numbers to each other to write as well) as they have to think about the place value as the number is not being described in order, on purpose

Bonds of 10 and bonds of other numbers up to 20 and calculating (addition and subtraction):

Teachers need to focus more on teaching the bonds of ALL the numbers between 1 and 9 and then for them to know how this is related to bonds of all teens numbers.

- Making bonds of ten using towers of linking cubes (When finding the opposite bond have the original amount in a tower to compare it with) - Have feely bag for bonds represented by multilink ('I can feel a 6 so I need a 3 to make 9' etc.)
- Bunny ears to show and investigate number bonds of other single digit numbers
- Set challenges for children to build a concept with the equipment to demonstrate how it works (i.e. 'Show me how that addition calculation works with cubes.'; 'Show me how to find the difference using those towers': 'Show me how you know that if 3x4=12 you also know that 12÷4=3 using these cubes'; etc.)
- The equals sign means a balance so be showing calculations from the beginning in different orders (but we still read the algorithm from left to right) and the use of a see-saw and balance scales help with this ideas. Ensure that children, from the point of beginning to write calculations, see and write them in a variety of ways and that they fully understand that the equals sign is a balance (i.e. 5x4=20 is also 20=4x5 and also that you can have 2x10=4x5 etc.)
- The hundred-flat supports children with recognising compliments of 100 i.e. why the tens number is equivalent to nine tens
- Discussing most appropriate method for subtraction calculations i.e. Will we 'take away' or 'find the difference'? When is each method most appropriate
- Ensure children can recognise which strategy is appropriate for which calculation by teaching them to study the numbers before they start to calculate:

expression	**Leads to the use of the following skill as opposed to counting back/on in ones**
76 - 6	Partitioning the number into tens and ones and 'taking the 6 off'
102 - 96	Finding the difference/distance by counting up from the smallest number
93 - 5	Know that 5=3+2 so able to do this by taking off the 3 to get to the previous multiple of ten and then take off the 2 (because that is what's left of the 5 that hasn't yet been subtracted). I can use bonds of 10 to know that if I subtract 2 from a multiple of 10 then the units value will be 8.
57 - 47	Recognise that these are 10 apart because the ones digits are the same as each other on both numbers and the tens digit is one ten apart.
42 - 26	Only partition the second number and keep the larger number whole. Take off the tens value by counting back in tens then take off the ones value by using the number bonds method

Fractions, decimals, percentages:

Teach these in any order as some children understand percentages more readily.

- Building fractions of amounts using towers of 12 cubes
- Discussion with the children of what fractions are and finding the most appropriate vocabulary to use when describing them
- Unpicking children's understanding of doubling and how this relates to their understanding of halving
- Using a variety of challenges and objects to find fractions of, to not limit their understanding of the concept, including cubes to work out equivalent fractions
- Physical activities that involve pacing out fractions on a long rope to support with ordering fractions
- There needs to be less emphasis on doubling as an addition and more on the idea that it is two groups of the same amount being combined as in scaling (a subtle difference but rather than recording as 3+3 it would be recorded as 3 x2 or 'double 3') to ensure that when the children use an inverse they don't try and subtract and rather, in this case as it is fractions, *share* into two equal pieces/groups/parts
- Explore finding fractions of a variety of objects to deepen their understanding of what fractions are, i.e. equilateral triangle, a circle with an arc cut out, a £2 coin, a potato which is irregular in shape, etc. – this isn't the only a picture of quarters 
- Use very specific vocabulary and descriptions when discussing fractions, i.e. 'one quarter is one part/group out of four equal parts/groups'
- Focus on fractions other than half and quarter as early as possible
- Work on the understanding of the fraction notation, i.e. that 7/7 is a whole as well as 2/2, 5/5, etc. also explore the names of the fractions (half as opposed to logically calling it a '2^{nd}' and quarter as opposed to logically calling it a '4^{th}') as well as describing the fraction as the top number being 'out of' the bottom number like a test result (this relates to the concept of proportion)
- Using the 100-flat from the base 10 blocks to show a picture of 100% as well as £1 and then one unit (each individual part of the 100-flat represented 1/100 as well as 1% and the 'ten stick' represented 10% as well as 1/10). The 100 bead string was also used in a similar way with each bead representing 1/100 and each group of 10 beads representing 1/10.
- Using towers of different colours to understand equivalent fractions i.e. a tower of three cubes, two red and one blue, where the blue is 'one out of three'. We write this as 1/3. Now imagine each cube is worth 2 then we write 2/6, each cube is now worth 3 so it is 3/6. etc. Then explore the numerical relationships.

Multiplication and division:

When linking multiplication and division, division has to be taught as 'grouping' rather than sharing.

- Use arrays to investigate factors (also looked at square numbers and prime numbers during this work with the same images as well as inverses for x and ÷)
- Repeated addition needs to be taught with a stronger link to multiplication through the use of arrays; times tables facts should be displayed and investigated through the use of arrays
- Using place value grids to demonstrate how digits move left when being multiplied by ten and exchanging equipment to represent a value in units for the equivalent value in tens to show how it gets ten times bigger (this was then related to other numbers and other algorithms i.e. ÷ by 10 and 100 as well as x by 10 and 100 and included decimal numbers up to two places)
- Division needs to be 'demystified' and children need to be introduced to the concept at the same time as counting in groups (also when we are sharing out which is connected to fractions); also the sign for division as early as when we introduce the multiplication sign
- When teachers and children are working on division facts (relating to multiplication facts) then the description of 'a quarter of 32 is 8' as well as '32 divided by 4 is 8' should also be used to ensure that the children link finding fractions of amounts to division and don't think of them as separate concepts
- Link modelling counting in groups with cubes to how it looks on a number line and use a partial jump to represent a remainder (it also looks like the letter 'r')
- Ensure children are meeting numbers when dividing that leave remainders (even if we just call them 'left over' at the start) as soon as they begin exploring counting in groups linked to dividing to not limit their understanding of how to divide into groups
- When asking the children to find patterns when investigating, ensure they focus on the pattern which is important i.e. when finding the equivalent coin facts the children were using the knowledge that 1 x, 10x and 100x the divisor are related and 2x, 20x and 200x the divisor are also related to each other as well as 5x, 50x and 500x as well. For an example of how to find 'coin facts' see over:

Coin facts are the key facts needed to support with division so-called because they are the same as the coin values and it helps with remembering the most useful facts. Start with the divisor multiplied by 1 and then multiply this answer by 10 and then 100 (1000 if needed). Double the one-times to find the divisor multiplied by 2 then use this fact to find the 20, 200 and (if needed) 2000 times the divisor. Finally halve the amount which is ten-times the divisor to find five-times the divisor then use this fact to find 50, 500 and (if needed) 5000 times the divisor.

e.g.: 489÷7

Coin facts of 7	
(1x)	7
(2x)	14
(5x)	35
(10x)	70
(20x)	140
(50x)	350
(100x)	700
(200x)	1400
(500x)	3500

so, 489÷7 = 69 r.6